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Signal estimation with multiple delayed sensors using covariance information

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ABSTRACT

Recursive filtering and smoothing algorithms to estimate a signal from noisy measurements coming from multiple randomly delayed sensors, with different delay characteristics, are proposed. To design these algorithms an innovation approach is used, assuming that the state-space model of the signal is unknown and using only covariance information. To measure the precision of the proposed estimators formulas to calculate the filtering and smoothing error covariance matrices are also derived. The effectiveness of the estimators is illustrated by a numerical simulation example where a signal is estimated using observations from two randomly delayed sensors having different delay properties.

1. Introduction

Estimation of signals using data coming from sensors or measurement devices that may be randomly delayed is a significant problem in different application fields. For example, in engineering applications involving communication networks with a heavy network traffic, the measurements available may not be up-to-date, and this fact must be considered in the study of the signal estimation problem. An appropriate model for such situations with randomly varying delays consists of interpreting the delay as a stochastic process, whose statistical properties are included in the system description [1].

In the last years, the state estimation problem for system models with randomly varying delays has been widely investigated. Assuming full knowledge of the state-space model for the signal process to be estimated, we will mention the following papers, among others: Ray et al. [2] proposed a recursive linear filtering algorithm which modifies the conventional one to fit situations where the arrival of sensor data at the controller terminal may be randomly delayed. In [3], the state estimation problem for a model involving randomly varying bounded sensor delays is treated by reformulating it as an estimation problem in systems with stochastic parameters. More recently, Su and Lu [4] have designed an extended Kalman filtering algorithm which provides optimal estimates of interconnected network states for systems in which some or all measurements are delayed. Matveev and Savkin [5] have proposed a recursive minimum variance state estimator in linear discrete-time partially observed systems perturbed by white noises, when the observations are transmitted via communication channels with random transmission times and various signal measurements may incur independent delays. Wang et al. [6] have designed a robust linear filter for linear uncertain discrete-time stochastic systems with randomly varying sensor delay.

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As indicated previously, the study performed in all these papers is based on the knowledge of the signal state-space model; however, this information is not available in many practical situations and the signal estimation problem must be addressed using another type of information. Assuming knowledge of the covariance functions of the processes involved in the observation equation and mutual independence of such processes, the signal estimation problem from randomly delayed observations has been addressed, for instance, in [7], where a linear filtering and fixed-point smoothing algorithm from randomly delayed observations is designed; more recently, this study has been generalized in [8], where a recursive algorithm for the second-order polynomial filter and fixed-point smoother is proposed. The linear filtering and smoothing problems have been also addressed from a more general observation model, involving correlated signal and additive noise, in [9,10].

Although all the papers mentioned above involve systems with randomly delayed sensors, their major handicap is that all the sensors are assumed to have the same delay characteristics. Recently, Hounkpevi and Yaz [11] have generalized this situation by considering multiple delayed sensors with different delay characteristics and, using the state-space model, the linear minimum variance state filter have been deduced for this new model. In the current paper, this general situation is considered and the least-squares linear filtering and smoothing (fixed-point and fixed-interval) problems are addressed assuming that the state-space model of the signal to be estimated is not known, but only the covariance functions of the signal and noise, as well as the parameters of the Bernoulli variables modeling the delays, are available. Therefore the proposed algorithms are applicable not only to signal processes that can be estimated by the conventional formulation using the state-space model but also to those for which a realization of the state-space model is not available.

2. Problem statement

In this section, the least-squares (LS) linear estimation problem of a n-dimensional discrete-time random signal, z_k, from noisy measurements coming from multiple sensors which are one-step randomly delayed, with different delay characteristics, is formulated. First, the observation model for these measurements and the hypotheses about the signal and noise processes are described.

2.1. Delayed observation model

Consider m scalar sensors whose real measurements, $\widetilde{y}_k^i$, of the signal, z_k, are perturbed by additive noise vectors v_k^i; that is,

$$\widetilde{y}_k^i = H_k^i z_k + v_k^i, \quad k \geqslant 1,\ i = 1, \ldots, m. \tag{1}$$

Assume that at time $k = 1$ the real measurements, $\widetilde{y}_1^i$, are always available for the estimation, but at each time $k > 1$ the available measurements coming from each sensor may be randomly delayed by one sampling period, according to different delay characteristics. It is also supposed that the ith sensor is delayed independently of the others and that a delay at time k is independent of a delay at time s. Therefore, if $\{\gamma_k^i;\ k > 1\}$, $i = 1, \ldots, m$, denote mutually independent sequences of independent Bernoulli random variables with $P[\gamma_k^i = 1] = p_k^i$, the available measurements of the ith sensor, y_k^i, are described by

$$y_k^i = \left(1 - \gamma_k^i\right)\widetilde{y}_k^i + \gamma_k^i \widetilde{y}_{k-1}^i, \quad k > 1; \qquad y_1^i = \widetilde{y}_1^i, \quad i = 1, \ldots, m. \tag{2}$$

From (2) it is clear that, if $\gamma_k^i = 1$, which occurs with probability p_k^i, then $y_k^i = \widetilde{y}_{k-1}^i$ and the measurement of the ith sensor is delayed by one sampling period; otherwise, if $\gamma_k^i = 0$, then $y_k^i = \widetilde{y}_k^i$, which means that the measurement is up-to-date with probability $1 - p_k^i$. Therefore, the variables $\{\gamma_k^i;\ k > 1\}$ model the random delay of the ith sensor and the values $\{p_k^i,\ k > 1\}$ represent the probabilities of delay in the measurements of the ith sensor.

In applications of communication networks, the noises $\{\gamma_k^i;\ k > 1\}$ usually represent the random delays from sensors to controller and the assumption of one-step sensor delay is based on the supposition that the induced data latency from the sensors to the controller is restricted so as not to exceed the sampling period. The delayed model considered in [7] covers those situations in which all the sensors present the same delay characteristics ($\gamma_k^i = \gamma_k$, $\forall i$), while the current delayed model considers measurements from multiple sensors featuring different random delay characteristics.

To treat the LS linear estimation problem of the signal based on the randomly delayed observations (2), the following hypotheses about the signal and noise processes are assumed:

(I) The n-dimensional signal process $\{z_k;\ k \geqslant 1\}$ has zero mean and its autocovariance function, $K_{k,s}^z = E[z_k z_s^T]$, is expressed in a semi-degenerate kernel form, $K_{k,s}^z = A_k B_s^T$, $s \leqslant k$, where A and B are known $n \times M$ matrix functions.

(II) For $i = 1, \ldots, m$, the scalar sensor noises, $\{v_k^i;\ k \geqslant 1\}$, are zero-mean white sequences with known variances, $\mathrm{Var}[v_k^i] = R_k^i$, $\forall k \geqslant 1$.

(III) For $i = 1, \ldots, m$, the processes modeling the random delay of the sensors, $\{\gamma_k^i;\ k > 1\}$ are sequences of independent Bernoulli random variables with known probabilities, $P[\gamma_k^i = 1] = p_k^i$, $\forall k > 1$.

(IV) The signal process, $\{z_k;\ k \geqslant 1\}$, and the noise processes, $\{v_k^i;\ k \geqslant 1\}$ and $\{\gamma_k^i;\ k > 1\}$, for $i = 1, \dots, m$, are mutually independent.

To simplify the notation, (1) and (2) are rewritten in a compact form as follows:

$$\widetilde{Y}_k = H_k z_k + V_k, \quad k \geqslant 1, \tag{3}$$

$$Y_k = \left(I_m - D_k^{\gamma}\right)\widetilde{Y}_k + D_k^{\gamma}\widetilde{Y}_{k-1}, \quad k > 1; \qquad Y_1 = \widetilde{Y}_1, \tag{4}$$

where $\widetilde{Y}_k = (\widetilde{y}_k^1, \dots, \widetilde{y}_k^m)^T$, $H_k = ((H_k^1)^T, \dots, (H_k^m)^T)^T$, $V_k = (v_k^1, \dots, v_k^m)^T$, $Y_k = (y_k^1, \dots, y_k^m)^T$, $D_k^{\gamma} = \mathrm{Diag}(\gamma_k^1, \dots, \gamma_k^m)$ and I_m is the $m \times m$ identity matrix.

Clearly, from hypotheses (II)–(IV), the following properties hold:

(i) The m-dimensional process $\{V_k;\ k \geqslant 1\}$ is a zero-mean white sequence with autocovariance function $E[V_k V_k^T] = R_k$, where $R_k = \mathrm{Diag}(R_k^1, \dots, R_k^m)$.
(ii) The random matrices $\{D_k^{\gamma};\ k > 1\}$ are independent and $E[D_k^{\gamma}] = D_k^p$, with $D_k^p = \mathrm{Diag}(p_k^1, \dots, p_k^m)$.
(iii) The random vectors $\{\gamma_k = (\gamma_k^1, \dots, \gamma_k^m)^T,\ k > 1\}$ have mean $E[\gamma_k] = p_k = (p_k^1, \dots, p_k^m)^T$ and the correlation functions of the random vectors γ_k and $\mathbf{1} - \gamma_k$ ($\mathbf{1} = (1, \dots, 1)^T$ is the $m \times 1$ ones vector) are

$$E\left[\gamma_k \gamma_k^T\right] = P_k^p, \qquad E\left[(\mathbf{1} - \gamma_k)(\mathbf{1} - \gamma_k)^T\right] = P_k^{\mathbf{1}-p}, \qquad E\left[\gamma_k(\mathbf{1} - \gamma_k)^T\right] = P_k^{p,\mathbf{1}-p},$$

where

$$P_k^a = \begin{pmatrix} a_k^1 & a_k^1 a_k^2 & \cdots & a_k^1 a_k^m \\ a_k^1 a_k^2 & a_k^2 & \cdots & a_k^2 a_k^m \\ \vdots & \vdots & \ddots & \vdots \\ a_k^1 a_k^m & a_k^2 a_k^m & \cdots & a_k^m \end{pmatrix}, \qquad P_k^{a,b} = \begin{pmatrix} 0 & a_k^1 b_k^2 & \cdots & a_k^1 b_k^m \\ a_k^1 b_k^2 & 0 & \cdots & a_k^2 b_k^m \\ \vdots & \vdots & \ddots & \vdots \\ a_k^1 b_k^m & a_k^2 b_k^m & \cdots & 0 \end{pmatrix}$$

with $a_k = (a_k^1, \dots, a_k^m)^T$ and $b_k = (b_k^1, \dots, b_k^m)^T$ the vectors p_k and $\mathbf{1} - p_k$.
(iv) $\{z_k;\ k \geqslant 1\}$, $\{V_k;\ k \geqslant 1\}$ and $\{D_k^{\gamma};\ k > 1\}$ are mutually independent.

2.2. Linear LS estimation problem

Our aim is to address the LS linear estimation problem of the signal, z_k, based on the randomly delayed observations $\{Y_1, \dots, Y_L\}$, with $L \geqslant k$, given in (4); more specifically, recursive algorithms for the filtering ($L = k$), fixed-point (k fixed and $L > k$) and fixed-interval (L fixed and $k < L$) smoothing problems will be derived.

As known, this estimator is the orthogonal projection of the vector z_k onto $\mathcal{L}(Y_1, \dots, Y_L)$, the linear space spanned by $\{Y_1, \dots, Y_L\}$; so the Orthogonal Projection Lemma (OPL) states that the estimator, $\widehat{z}_{k/L}$, is the only linear combination satisfying the orthogonality property

$$E\left[(z_k - \widehat{z}_{k/L})Y_s^T\right] = 0, \quad s = 1, \dots, L.$$

Due to the fact that generally the observations are nonorthogonal vectors, we will use an innovation approach [12], consisting of transforming the observation process $\{Y_k;\ k \geqslant 1\}$ to an equivalent process (innovation process) of orthogonal vectors $\{\nu_k;\ k \geqslant 1\}$, equivalent in the sense that each set $\{\nu_1, \dots, \nu_L\}$ spans the same linear subspace that $\{Y_1, \dots, Y_L\}$; that is, $\mathcal{L}(\nu_1, \dots, \nu_L) = \mathcal{L}(Y_1, \dots, Y_L)$.

The innovation process is constructed by the *Gram-Schmidt orthogonalization procedure*, using an inductive reasoning. Starting with $\nu_1 = Y_1$, the projection of the next observation, Y_2, onto $\mathcal{L}(\nu_1)$ is given by $\widehat{Y}_{2/1} = E[Y_2 \nu_1^T](E[\nu_1 \nu_1^T])^{-1}\nu_1$; then, the vector $\nu_2 = Y_2 - \widehat{Y}_{2/1}$ is orthogonal to ν_1 and clearly $\mathcal{L}(\nu_1, \nu_2) = \mathcal{L}(Y_1, Y_2)$. Let $\{\nu_1, \dots, \nu_{k-1}\}$ be the set of orthogonal vectors satisfying $\mathcal{L}(\nu_1, \dots, \nu_{k-1}) = \mathcal{L}(Y_1, \dots, Y_{k-1})$. If now we have an additional observation Y_k, we project it onto $\mathcal{L}(\nu_1, \dots, \nu_{k-1})$; the orthogonality property allows us to find the projection by separately projecting onto each of the previous orthogonal vectors, that is,

$$\widehat{Y}_{k/k-1} = \sum_{j=1}^{k-1} E\left[Y_k \nu_j^T\right]\left(E\left[\nu_j \nu_j^T\right]\right)^{-1}\nu_j;$$

so the next vector, $\nu_k = Y_k - \widehat{Y}_{k/k-1}$, is orthogonal to the previous ones and $\mathcal{L}(\nu_1, \dots, \nu_k) = \mathcal{L}(Y_1, \dots, Y_k)$.

Note that the projection $\widehat{Y}_{k/k-1}$ is the part of the observation Y_k that is determined by knowledge of $\{Y_1, \dots, Y_{k-1}\}$; thus the remainder vector $\nu_k = Y_k - \widehat{Y}_{k/k-1}$ can be regarded as the "new information" or the "innovation" provided by Y_k, and the process $\{\nu_k;\ k \geqslant 1\}$ as the innovation process associated with $\{Y_k;\ k \geqslant 1\}$. The causal and causally invertible linear relation existing between the observation and innovation processes makes the innovation process unique.

Next, taking into account that the innovations constitute a white process, we derive a general expression for the LS linear estimator of the signal, $\widehat{z}_{k/L}$. Replacing $\{Y_1, \ldots, Y_L\}$ by the equivalent set of orthogonal vectors $\{\nu_1, \ldots, \nu_L\}$, the signal estimator is

$$\widehat{z}_{k/L} = \sum_{j=1}^{L} h_{k,j}\nu_j,$$

where the impulse-response function $h_{k,j}$, $j = 1, \ldots, L$ is calculated from the orthogonality property,

$$E\big[(z_k - \widehat{z}_{k/L})\nu_s^T\big] = 0, \quad s \leqslant L,$$

which leads to the Wiener-Hopf equation:

$$E\big[z_k\nu_s^T\big] = \sum_{j=1}^{L} h_{k,j}E\big[\nu_j\nu_s^T\big], \quad s \leqslant L.$$

Due to the whiteness of the innovation process, $E[\nu_j\nu_s^T] = 0$ for $j \neq s$ and the Wiener-Hopf equation is expressed as

$$E\big[z_k\nu_s^T\big] = h_{k,s}E\big[\nu_s\nu_s^T\big], \quad s \leqslant L;$$

consequently,

$$h_{k,s} = E\big[z_k\nu_s^T\big]\big(E\big[\nu_s\nu_s^T\big]\big)^{-1}, \quad s \leqslant L$$

and, therefore, the following general expression for the LS linear estimator of the signal is obtained

$$\widehat{z}_{k/L} = \sum_{j=1}^{L} S_{k,j}\Pi_j^{-1}\nu_j \tag{5}$$

where $S_{k,j} = E[z_k\nu_j^T]$ and $\Pi_j = E[\nu_j\nu_j^T]$.

Starting from this general expression, it is clear that the linear fixed-point smoothers, $\widehat{z}_{k/L}$, $L > k$, can be recursively calculated as

$$\widehat{z}_{k/L} = \widehat{z}_{k/L-1} + S_{k,L}\Pi_L^{-1}\nu_L, \quad L > k, \tag{6}$$

with initial condition given by the linear filter, $\widehat{z}_{k/k}$.

Again, taking into account expression (5), the fixed-interval smoothers, $\widehat{z}_{k/L}$, $k < L$, can be obtained from the filter, $\widehat{z}_{k/k}$, by adding the remaining terms of the sum in (5)

$$\widehat{z}_{k/L} = \widehat{z}_{k/k} + \sum_{j=k+1}^{L} S_{k,j}\Pi_j^{-1}\nu_j, \quad k < L. \tag{7}$$

Therefore, our first purpose will be to design an algorithm for the filter and, afterwards, the fixed-point and fixed-interval smoothing algorithms will be derived from the filter and expressions (6) and (7), respectively.

3. Linear filtering algorithm

In view of expression (5) for $L = k$, to obtain the filter, $\widehat{z}_{k/k}$, it is necessary to determine the matrices $S_{k,j} = E[z_k\nu_j^T]$, the innovations ν_j and their covariances matrices Π_j, for $j \leqslant k$.

We will start by obtaining an explicit formula for the innovations, $\nu_j = Y_j - \widehat{Y}_{j/j-1}$, or equivalently for $\widehat{Y}_{j/j-1}$, the one-stage predictor of Y_j, which, denoting $T_{j,i} = E[Y_j\nu_i^T]$, is given by

$$\widehat{Y}_{j/j-1} = \sum_{i=1}^{j-1} T_{j,i}\Pi_i^{-1}\nu_i, \quad j \geqslant 2; \qquad \widehat{Y}_{1/0} = 0. \tag{8}$$

First, we obtain $T_{j,i} = E[Y_jY_i^T] - E[Y_j\widehat{Y}_{i/i-1}^T]$, for $1 \leqslant i \leqslant j-1$ and $j \geqslant 2$. Using (I) and properties (i), (ii) and (iv), we have

$$E\big[Y_jY_i^T\big] = \begin{cases} G_{A_j}G_{B_i}^T, & 1 < i < j-1, \\ G_{A_j}G_{B_{j-1}}^T + D_j^p R_{j-1}(I_m - D_{j-1}^p), & i = j-1, \end{cases}$$

where

$$G_{X_j} = \big(I_m - D_j^p\big)H_jX_j + D_j^pH_{j-1}X_{j-1}, \quad X = A, B. \tag{9}$$

Substituting these expectations in $T_{j,i} = E[Y_j Y_i^T] - E[Y_j \widehat{Y}_{i/i-1}^T]$ and using (8) for $\widehat{Y}_{i/i-1}$, we can write, for $i > 1$,

$$T_{j,i} = \begin{cases} G_{A_j} G_{B_i}^T - \sum_{l=1}^{i-1} T_{j,l} \Pi_l^{-1} T_{i,l}^T, & 1 < i < j-1, \\ G_{A_j} G_{B_{j-1}}^T - \sum_{l=1}^{j-2} T_{j,l} \Pi_l^{-1} T_{j-1,l}^T + D_j^p R_{j-1}(I_m - D_{j-1}^p), & i = j-1 \end{cases}$$

and, since $\nu_1 = Y_1$, it is easy to see that

$$T_{j,1} = E\big[Y_j Y_1^T\big] = \begin{cases} G_{A_2} B_1^T H_1^T + D_2^p R_1, & j = 2, \\ G_{A_j} B_1^T H_1^T, & j > 2. \end{cases}$$

This expression for $T_{j,i}$, $1 \leqslant j \leqslant i-1$, guarantees that for $j \geqslant 2$,

$$T_{j,i} = \begin{cases} G_{A_j} J_i, & 1 \leqslant i < j-1, \\ G_{A_j} J_{j-1} + F_{j-1}, & i = j-1, \end{cases} \tag{10}$$

where J is a function satisfying

$$J_i = G_{B_i}^T - \sum_{l=1}^{i-1} J_l \Pi_l^{-1} T_{i,l}^T, \quad i \geqslant 2; \qquad J_1 = B_1^T H_1^T, \tag{11}$$

and

$$F_{j-1} = D_j^p R_{j-1}\big(I_m - D_{j-1}^p\big), \quad j > 2; \qquad F_1 = D_2^p R_1. \tag{12}$$

Substituting (10) into (8), we obtain

$$\widehat{Y}_{j/j-1} = G_{A_j} O_{j-1} + F_{j-1} \Pi_{j-1}^{-1} \nu_{j-1}, \quad j \geqslant 2, \tag{13}$$

and, consequently,

$$\nu_j = Y_j - G_{A_j} O_{j-1} - F_{j-1} \Pi_{j-1}^{-1} \nu_{j-1}, \quad j \geqslant 2,$$

where

$$O_j = \sum_{i=1}^{j} J_i \Pi_i^{-1} \nu_i, \quad j \geqslant 1; \qquad O_0 = 0, \tag{14}$$

which can be recursively obtained as

$$O_j = O_{j-1} + J_j \Pi_j^{-1} \nu_j, \quad j \geqslant 1; \qquad O_0 = 0. \tag{15}$$

Now, to obtain the matrix J_j we substitute (10) into (11), having that

$$J_j = G_{B_j}^T - r_{j-1} G_{A_j}^T - J_{j-1} \Pi_{j-1}^{-1} F_{j-1}, \quad j \geqslant 2; \qquad J_1 = B_1^T H_1^T,$$

with $r_j = E[O_j O_j^T]$ recursively obtained from

$$r_j = r_{j-1} + J_j \Pi_j^{-1} J_j^T, \quad j \geqslant 1; \qquad r_0 = 0.$$

Next, the innovation covariance matrix $\Pi_j = E[\nu_j \nu_j^T] = E[Y_j Y_j^T] - E[\widehat{Y}_{j/j-1} \widehat{Y}_{j/j-1}^T]$ will be calculated.

From (I) and properties (i)–(iv),

$$\begin{aligned} E\big[Y_j Y_j^T\big] = {} & P_j^{\mathbf{1}-p} \circ \big[H_j A_j B_j^T H_j^T + R_j\big] + P_j^p \circ \big[H_{j-1} A_{j-1} B_{j-1}^T H_{j-1}^T + R_{j-1}\big] \\ & + P_j^{\mathbf{1}-p,p} \circ \big[H_j A_j B_{j-1}^T H_{j-1}^T\big] + P_j^{p,\mathbf{1}-p} \circ \big[H_{j-1} B_{j-1} A_j^T H_j^T\big], \quad j \geqslant 2, \end{aligned}$$

where $\circ$ denotes the Hadamard product ($[C \circ D]_{ij} = C_{ij} D_{ij}$).

Next, using (8) for $\widehat{Y}_{j/j-1}$ with (10) for $T_{j,i}$, we obtain

$$\begin{aligned} \Pi_j = {} & P_j^{\mathbf{1}-p} \circ \big[H_j A_j B_j^T H_j^T + R_j\big] + P_j^p \circ \big[H_{j-1} A_{j-1} B_{j-1}^T H_{j-1}^T + R_{j-1}\big] \\ & + P_j^{\mathbf{1}-p,p} \circ \big[H_j A_j B_{j-1}^T H_{j-1}^T\big] + P_j^{p,\mathbf{1}-p} \circ \big[H_{j-1} B_{j-1} A_j^T H_j^T\big] \\ & - G_{A_j}\big[G_{B_j}^T - J_j\big] - F_{j-1} \Pi_{j-1}^{-1} \big[G_{A_j} J_{j-1} + F_{j-1}\big]^T, \quad j \geqslant 2, \\ \Pi_1 = {} & H_1 A_1 B_1^T H_1^T + R_1. \end{aligned}$$

Once the innovations ν_j and their covariances matrices Π_j have been specified, we must calculate $S_{k,j} = E[z_k \nu_j^T] = E[z_k Y_j^T] - E[z_k \widehat{Y}_{j/j-1}^T]$, for $1 \leqslant j \leqslant k$. From (I) and properties (ii) and (iv), it is easy to see that $E[z_k Y_1^T] = A_k B_1^T H_1^T$ and $E[z_k Y_j^T] = A_k G_{B_j}^T$ for $j > 1$; then using (8) for $\widehat{Y}_{j/j-1}$ in $E[z_k \widehat{Y}_{j/j-1}^T]$, we have that

$$S_{k,j} = A_k J_j, \quad 1 \leqslant j \leqslant k, \tag{16}$$

where J is the function satisfying (11). Hence, substituting $S_{k,j} = A_k J_j$ in (5) for $L = k$ and using (14), it is immediate that $\widehat{z}_{k/k} = A_k O_k$.

The above results are summarized in the following *linear filtering algorithm*:

The linear filter, $\widehat{z}_{k/k}$, of the signal z_k is obtained as

$$\widehat{z}_{k/k} = A_k O_k, \quad k \geqslant 1, \tag{17}$$

where the vectors O_k are recursively calculated from

$$O_k = O_{k-1} + J_k \Pi_k^{-1} \nu_k, \quad k \geqslant 1; \qquad O_0 = 0,$$

and the matrix J_k is given by

$$J_k = G_{B_k}^T - r_{k-1} G_{A_k}^T - J_{k-1} \Pi_{k-1}^{-1} F_{k-1}, \quad k \geqslant 2; \qquad J_1 = B_1^T H_1^T,$$

with $r_k = E[O_k O_k^T]$ recursively obtained from

$$r_k = r_{k-1} + J_k \Pi_k^{-1} J_k^T, \quad k \geqslant 1; \qquad r_0 = 0.$$

The innovation, ν_k, satisfies

$$\nu_k = Y_k - G_{A_k} O_{k-1} - F_{k-1} \Pi_{k-1}^{-1} \nu_{k-1}, \quad k \geqslant 2; \qquad \nu_1 = Y_1$$

and Π_k, the innovation covariance matrix, is given by

$$\begin{aligned}
\Pi_k &= P_k^{\mathbf{1}-p} \circ \left[H_k A_k B_k^T H_k^T + R_k\right] + P_k^{p} \circ \left[H_{k-1} A_{k-1} B_{k-1}^T H_{k-1}^T + R_{k-1}\right] \\
&\quad + P_k^{\mathbf{1}-p,p} \circ \left[H_k A_k B_{k-1}^T H_{k-1}^T\right] + P_k^{p,\mathbf{1}-p} \circ \left[H_{k-1} B_{k-1} A_k^T H_k^T\right] \\
&\quad - G_{A_k}\left[G_{B_k}^T - J_k\right] - F_{k-1} \Pi_{k-1}^{-1}\left[G_{A_k} J_{k-1} + F_{k-1}\right]^T, \quad k \geqslant 2, \\
\Pi_1 &= H_1 A_1 B_1^T H_1^T + R_1.
\end{aligned}$$

The matrices G_{A_k}, G_{B_k} and F_{k-1} are given in (9) *and* (12), *respectively.*

The accuracy of the LS linear filter is measured by the filtering error covariance matrices

$$\Sigma_{k/k} = E\left[\{z_k - \widehat{z}_{k/k}\}\{z_k - \widehat{z}_{k/k}\}^T\right].$$

Since the error $z_k - \widehat{z}_{k/k}$ is orthogonal to the estimator $\widehat{z}_{k/k}$, it is clear that

$$\Sigma_{k/k} = K_{k,k}^z - E\left[\widehat{z}_{k/k} \widehat{z}_{k/k}^T\right].$$

Then, hypothesis (I) on the model and expression (17) for the filter lead to the following formula for the *filtering error covariance matrices*

$$\Sigma_{k/k} = A_k B_k^T - A_k r_k A_k^T, \quad k \geqslant 1. \tag{18}$$

4. Linear smoothing algorithms

In this section, we present recursive algorithms for the LS linear smoothers, $\widehat{z}_{k/L}$, $L \geqslant k$. More precisely, the fixed-point and fixed-interval smoothing problems are addressed. In the fixed-point smoothing problem k is fixed and recursions for increasing L are proposed (Section 4.1), while in the fixed-interval smoothing problem the number of available observations, L, is fixed and recursions in k are established (Section 4.2). As indicated in Section 2, these algorithms will be derived by starting from the filter and expressions (6) and (7), respectively.

4.1. Fixed-point smoothing algorithm

To calculate the fixed-point smoothing estimators, $\widehat{z}_{k/L}$, for $L > k$ (k fixed), from (6) a recursive relation in L must be obtained for $S_{k,L} = E[z_k \nu_L^T] = E[z_k Y_L^T] - E[z_k \widehat{Y}_{L/L-1}^T]$, $L \geqslant k$. Using (I) and properties (ii) and (iv), it is easy to see that $E[z_k Y_L^T] = B_k G_{A_L}^T$ for $L > k$, which, together with expression (13) for $j = L$, yields

$$S_{k,L} = B_k G_{A_L}^T - E\big[z_k O_{L-1}^T\big] G_{A_L}^T - S_{k,L-1}\Pi_{L-1}^{-1} F_{L-1}, \quad L > k,$$

and defining the function $E_{k,L} = E[z_k O_L^T]$, the following expression holds

$$S_{k,L} = (B_k - E_{k,L-1}) G_{A_L}^T - S_{k,L-1}\Pi_{L-1}^{-1} F_{L-1}, \quad L > k.$$

From (16), the initial condition for the expression above is clearly $S_{k,k} = A_k J_k$.

Next, using the recursive expression (15) for O_L, the following formula for $E_{k,L}$ is deduced

$$E_{k,L} = E_{k,L-1} + S_{k,L}\Pi_L^{-1} J_L^T, \quad L > k.$$

Its initial condition is $E_{k,k} = A_k r_k$; this expression is easily derived taking into account that, from the OPL, $E_{k,k} = E[z_k O_k^T] = E[\widehat{z}_{k/k} O_k^T]$ and using (17) and that $r_k = E[O_k O_k^T]$.

Summarizing these results, the following recursive *fixed-point smoothing algorithm* is obtained:

The fixed-point smoother $\widehat{z}_{k/L}$, with $L > k$, of the signal z_k is calculated as

$$\widehat{z}_{k/L} = \widehat{z}_{k/L-1} + S_{k,L}\Pi_L^{-1}\nu_L, \quad L > k,$$

with initial condition given by the filter, $\widehat{z}_{k/k}$, and

$$S_{k,L} = (B_k - E_{k,L-1}) G_{A_L}^T - S_{k,L-1}\Pi_{L-1}^{-1} F_{L-1}, \quad L > k; \qquad S_{k,k} = A_k J_k,$$

where the matrices $E_{k,L}$ satisfy the following recursive formula

$$E_{k,L} = E_{k,L-1} + S_{k,L}\Pi_L^{-1} J_L^T, \quad L > k; \qquad E_{k,k} = A_k r_k.$$

The filter $\widehat{z}_{k/k}$, the matrices G_{A_L}, F_L and J_L, the innovations ν_L and their covariance matrices Π_L are obtained from the linear filtering algorithm given in Section 3.

Following a similar reasoning to that used to derive the filtering error covariance matrices, but using now the recursive formula of the fixed-point smoother, the *fixed-point smoothing error covariance matrices*,

$$\Sigma_{k/L} = E\big[\{z_k - \widehat{z}_{k/L}\}\{z_k - \widehat{z}_{k/L}\}^T\big], \quad L > k$$

are recursively obtained as follows

$$\Sigma_{k/L} = \Sigma_{k/L-1} - S_{k,L}\Pi_L^{-1} S_{k/L}^T, \quad L > k,$$

with initial condition $\Sigma_{k/k}$, given in (18).

4.2. Fixed-interval smoothing algorithm

To obtain the fixed-interval smoother, $\widehat{z}_{k/L}$, for $k < L$, we must establish recursions in k when the number of available observations, L, is fixed. From (7), we start by obtaining the coefficients $S_{k,j} = [z_k \nu_j^T]$, for $j \geqslant k+1$. As in the previous section, the following expression holds

$$S_{k,j} = B_k G_{A_j}^T - E\big[z_k O_{j-1}^T\big] G_{A_j}^T - S_{k,j-1}\Pi_{j-1}^{-1} F_{j-1}, \quad j \geqslant k+1,$$

and, using (14) for O_{j-1}, we have

$$S_{k,j} = B_k G_{A_j}^T - \sum_{l=1}^{j-1} S_{k,l}\Pi_l^{-1} J_l^T G_{A_j}^T - S_{k,j-1}\Pi_{j-1}^{-1} F_{j-1}, \quad j \geqslant k+1.$$

Now, using (16) and that $r_k = \sum_{l=1}^{j-1} J_l \Pi_l^{-1} J_l^T$, this expression can be rewritten as follows

$$S_{k,j} = (B_k - A_k r_k) G_{A_j}^T - \sum_{l=k+1}^{j-1} S_{k,l}\Pi_l^{-1} J_l^T G_{A_j}^T - S_{k,j-1}\Pi_{j-1}^{-1} F_{j-1}, \quad j > k+1,$$

$$S_{k,k+1} = (B_k - A_k r_k) G_{A_{k+1}}^T - A_k J_k \Pi_k^{-1} F_k.$$

This relation guarantees that[1]

$$S_{k,j} = (B_k - A_k r_k \mid -A_k J_k)\Delta_{k,j}, \quad j \geqslant k+1,$$

where $\Delta_{k,j} = (\Delta^{(1)T}_{k,j} \mid \Delta^{(2)T}_{k,j})^T$ is given by

$$\Delta_{k,j} = \begin{pmatrix} G^T_{A_j} - \sum_{l=k+1}^{j-1} \Delta^{(1)}_{k,l} \Pi_l^{-1} J_l^T G^T_{A_j} - \Delta^{(1)}_{k,j-1} \Pi^{-1}_{j-1} F_{j-1} \\ -\sum_{l=k+1}^{j-1} \Delta^{(2)}_{k,l} \Pi_l^{-1} J_l^T G^T_{A_j} - \Delta^{(2)}_{k,j-1} \Pi^{-1}_{j-1} F_{j-1} \end{pmatrix}, \quad j > k+1;$$

$$\Delta_{k,k+1} = \left(G_{A_{k+1}} \mid F_k \Pi_k^{-1}\right)^T.$$

Therefore, from (7), if we define

$$q_{k/L} = \sum_{j=k+1}^{L} \Delta_{k,j} \Pi_j^{-1} \nu_j, \quad k < L; \qquad q_{L/L} = 0,$$

and

$$\Upsilon_k = (B_k - A_k r_k \mid -A_k J_k) \tag{19}$$

the following expression for the smoother $\widehat{z}_{k/L}$ is immediately obtained

$$\widehat{z}_{k/L} = \widehat{z}_{k/k} + \Upsilon_k q_{k/L}, \quad k < L.$$

Next, a backward recursive expression for $q_{k/L}$ will be derived. If we calculate the difference $\Delta_{k,j} - \Delta_{k+1,j}$, for $j \geqslant k+2$, and compare the expression obtained with the resulting one for $\Delta_{k+1,j}$, the following relation is deduced

$$\Delta_{k,j} = \begin{pmatrix} I_M - G^T_{A_{k+1}} \Pi^{-1}_{k+1} J^T_{k+1} & -G^T_{A_{k+1}} \\ -\Pi_k^{-1} F_k \Pi^{-1}_{k+1} J^T_{k+1} & -\Pi_k^{-1} F_k \end{pmatrix} \Delta_{k+1,j}, \quad j \geqslant k+2.$$

Using this relation, the following backward recursive formula for $q_{k/L}$ is easily obtained by separating the term corresponding to $j = k+1$ in the definition of $q_{k/L}$

$$q_{k/L} = \Xi_{k+1} \begin{pmatrix} q_{k+1/L} \\ \Pi^{-1}_{k+1} \nu_{k+1} \end{pmatrix}, \quad k < L$$

with the matrix Ξ_k given by

$$\Xi_k = \begin{pmatrix} I_M - G^T_{A_k} \Pi_k^{-1} J_k^T & -G^T_{A_k} & G^T_{A_k} \\ -\Pi^{-1}_{k-1} F_{k-1} \Pi_k^{-1} J_k^T & -\Pi^{-1}_{k-1} F_{k-1} & \Pi^{-1}_{k-1} F_{k-1} \end{pmatrix}. \tag{20}$$

These results are summarized in the following recursive *fixed-interval smoothing algorithm*:

The fixed-interval smoothing estimators $\widehat{z}_{k/L}$, for $k < L$, are calculated from

$$\widehat{z}_{k/L} = \widehat{z}_{k/k} + \Upsilon_k q_{k/L}, \quad k < L, \tag{21}$$

where, starting with $q_{L/L} = 0$, the vectors $q_{k/L}$ are backward recursively obtained from

$$q_{k/L} = \Xi_{k+1} \begin{pmatrix} q_{k+1/L} \\ \Pi^{-1}_{k+1} \nu_{k+1} \end{pmatrix}, \quad k < L,$$

with Υ_k and Ξ_k given in (19) *and* (20)*, respectively. The filter $\widehat{z}_{k/k}$, the matrices G_{A_k}, F_k and J_k, the innovations ν_k and their covariance matrices Π_k are obtained from the linear filtering algorithm given in Section* 3.

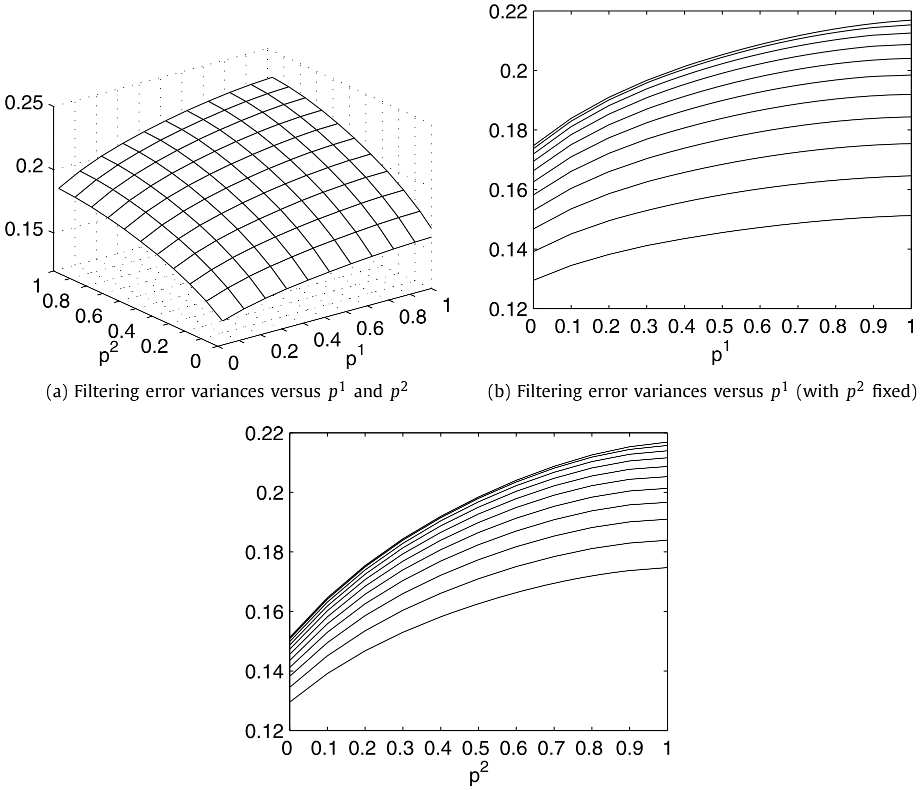
Finally, using expression (21) for the fixed-interval smoother, the following formula for the *fixed-interval smoothing error covariance matrices* is deduced

$$\Sigma_{k/L} = \Sigma_{k/k} - \Upsilon_k Q_{k/L} \Upsilon_k^T, \quad k < L,$$

where $\Sigma_{k/k}$ and Υ_k are given in (18) and (19), respectively. The matrices $Q_{k/L} = E[q_{k/L} q^T_{k/L}]$ satisfy the following recursive relation

$$Q_{k/L} = \Xi_{k+1} \begin{pmatrix} Q_{k+1/L} & 0 \\ 0 & \Pi^{-1}_{k+1} \end{pmatrix} \Xi^T_{k+1}, \quad k < L; \qquad Q_{L/L} = 0.$$

[1] $(U \mid V)$ denotes a partitioned matrix into two sub-matrices U and V.



(a) Filtering error variances versus p^1 and p^2

(b) Filtering error variances versus p^1 (with p^2 fixed)

(c) Filtering error variances versus p^2 (with p^1 fixed)

Fig. 1. Filtering error variances versus p^1 and p^2.

5. Example: scalar signal estimation

This section shows a numerical simulation example to illustrate the application of the recursive algorithms proposed in the paper. To show the effectiveness of the proposed estimators, a program in MATLAB has been run, simulating at each iteration the signal and the observed values and providing the filtering and smoothing estimates, as well as the corresponding error covariance matrices.

Consider a zero-mean scalar signal $\{z_k;\ k \geqslant 1\}$ with autocovariance function given by

$$K^z_{k,s} = 1.025641 \times 0.95^{k-s}, \quad s \leqslant k,$$

which is factorizable according to hypothesis (I) taking

$$A_k = 1.025641 \times 0.95^k, \qquad B_k = 0.95^{-k}.$$

For the simulation, the signal is supposed to be generated by the following first-order autoregressive model

$$z_{k+1} = 0.95 z_k + w_k$$

where $\{w_k;\ k \geqslant 1\}$ is a zero-mean white Gaussian noise with $\mathrm{Var}[w_k] = 0.1$, for all k.

Consider two sensors whose real measurements, $\widetilde{y}^i_k = z_k + v^i_k$, $i = 1, 2$, are perturbed by independent zero-mean white Gaussian noises, $\{v^i_k;\ k \geqslant 1\}$, with constant variances for all k, $\mathrm{Var}[v^1_k] = 0.5$ and $\mathrm{Var}[v^2_k] = 0.9$.

Now, according to the current model, assume that, at any time $k > 1$, the available measurement of the ith sensor, y^i_k, may be delayed by one sampling period with different delay characteristics; specifically, we consider that

$$y^i_k = \left(1 - \gamma^i_k\right)\widetilde{y}^i_k + \gamma^i_k \widetilde{y}^i_{k-1}, \quad k > 1; \qquad y^i_1 = \widetilde{y}^i_1, \quad i = 1, 2,$$

where $\{\gamma^i_k;\ k \geqslant 1\}$, $i = 1, 2$, are sequences of independent Bernoulli random variables with constant delay probabilities, $P[\gamma^i_k = 1] = p^i$, $\forall k > 1$.

First, we study the filtering error variances, $\Sigma_{k/k}$, when the delay probabilities p^1 and p^2 are varied from 0 to 1. It must be noted that such error variances stabilize around a constant value for k greater or equal to 10. For this reason, Fig. 1(a) displays the filtering error variances $\Sigma_{10/10}$ versus p^1 and p^2. Fig. 1(b), obtained by cutting the surface in Fig. 1(a) with the

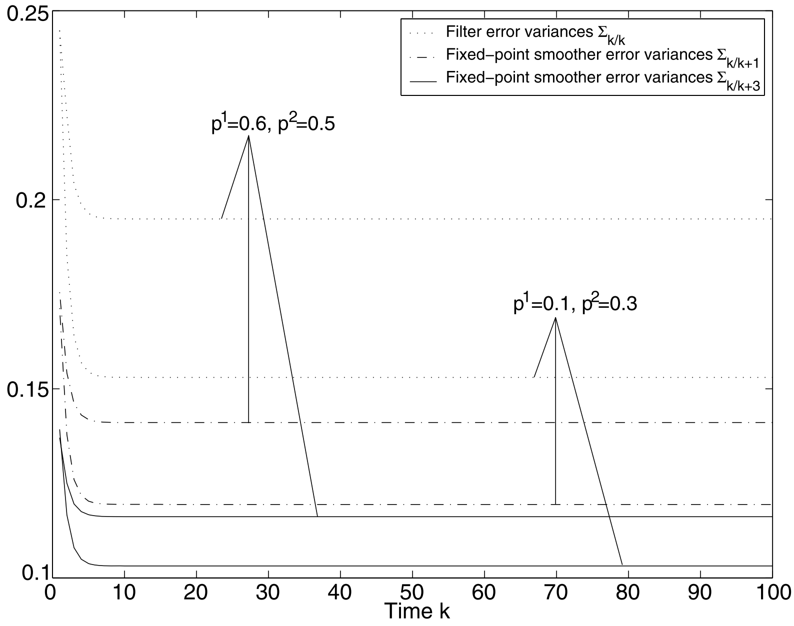


Fig. 2. Filtering error variances and fixed-point smoothing error variances, for the values $p^1 = 0.1$, $p^2 = 0.3$ and $p^1 = 0.6$, $p^2 = 0.5$.

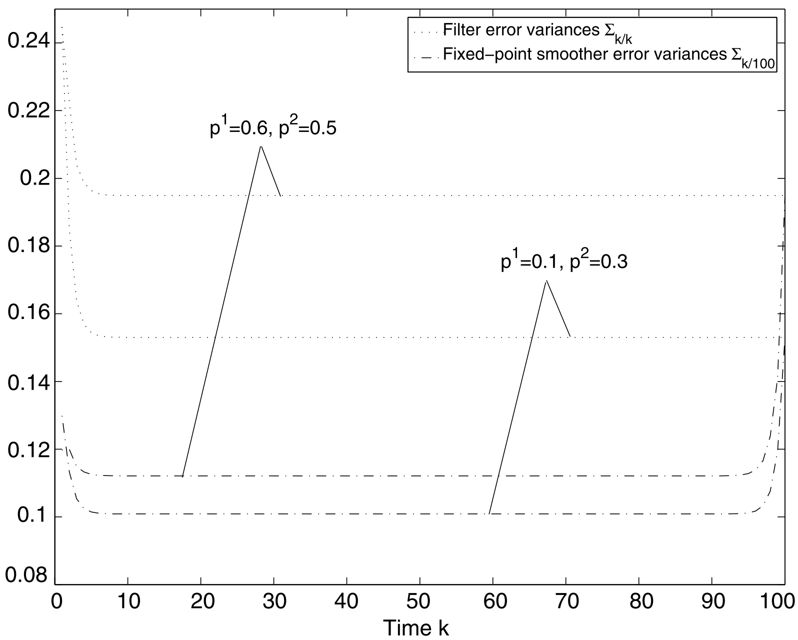


Fig. 3. Filtering error variances and fixed-interval smoothing error variances, for the values $p^1 = 0.1$, $p^2 = 0.3$ and $p^1 = 0.6$, $p^2 = 0.5$.

planes $p^2 = 0$, $p^2 = 0.1, \ldots, p^2 = 1$, shows the filtering error variances versus p^1. And Fig. 1(c) shows these variances versus p^2 (for constant values of p^1). From this figure it is gathered that, as the probability that the observations of both sensors are delayed increases, the filtering error variances become greater and, consequently, the performance of the estimators is worse.

Next, to compare the effectiveness of the proposed filtering and smoothing estimators, one hundred iterations of the respective algorithms have been performed, considering different values of the delay probabilities; on the one hand $p^1 = 0.1$, $p^2 = 0.3$ and, on the other, $p^1 = 0.6$, $p^2 = 0.5$. For these values, the filtering and smoothing estimates (fixed-point and fixed-interval), as well as the corresponding error variances, have been calculated.

Fig. 2 displays the filtering error variances, $\Sigma_{k/k}$, and the fixed-point smoothing error variances, $\Sigma_{k/k+2}$ and $\Sigma_{k/k+5}$. This figure shows, on the one hand, that the error variances corresponding to the smoothers are less than the filtering ones and,

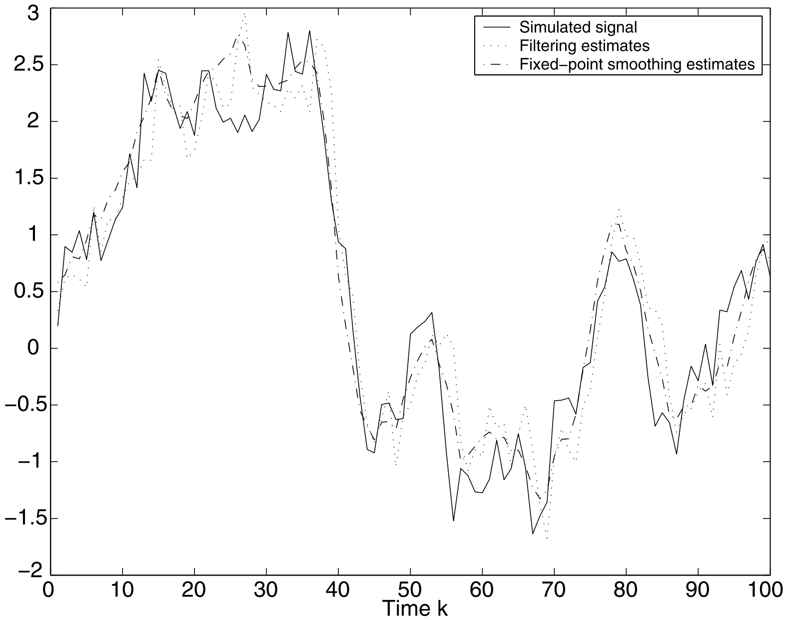


Fig. 4. Simulated signal, z_k, filtering estimates, $\widehat{z}_{k/k}$, and fixed-point smoothing estimates, $\widehat{z}_{k/k+3}$, for the values $p^1 = 0.1$ and $p^2 = 0.3$.

on the other, that as the delay probabilities of the sensors decreases, the error variances are smaller and consequently, the performance of the estimators is better, according to what was observed in Fig. 1. From this figure, it is also deduced that the accuracy of the smoother at each fixed-point k is better as the number of available observations increases.

For the same values of the delay probabilities, the fixed-interval smoothing error variances, $\Sigma_{k/100}$ are shown in Fig. 3 together with the filtering error variances, $\Sigma_{k/k}$. From this figure, similar considerations to those of Fig. 2 are inferred.

Finally, for the values $p^1 = 0.1$ and $p^2 = 0.3$, Fig. 4 shows a simulated signal together with the filtering estimates, $\widehat{z}_{k/k}$, and fixed-point smoothing estimates, $\widehat{z}_{k/k+3}$, while Fig. 5 shows a different simulated signal together with the filtering estimates and the fixed-interval smoothing estimates, $\widehat{z}_{k/100}$. Agreeing with the results obtained from Figs. 2 and 3, these figures show that the smoothing estimates follow the signal evolution better than the filtering ones.

On the other hand, since in the theoretical study we have considered that the delay probabilities at each sensor are time-variant, next we analyze, as an example, the filter performance when the delay probabilities at the first sensor are time-dependent. Fig. 6 shows the filtering error variances and the delay probabilities in the observations coming from the first sensor, p_k^1 (varying at each sampling time k), when p_k^2, as in Figs. 2 and 3, takes the fixed values $p^2 = 0.3$ and $p^2 = 0.5$, for all k. From this figure it is observed that, at each sampling time, the filtering error variance increases (respectively, decreases)–and, consequently, worse (respectively, better) estimations are obtained–as the delay probability in the observations coming from the first sensor is greater (respectively, smaller).

6. Concluding remark

In this paper, least-squares linear filtering and smoothing recursive algorithms are proposed to estimate signals from randomly delayed observations coming from multiple sensors with different delay characteristics. This is a realistic assumption in situations concerning sensor data that are transmitted over communication networks where, generally, multiple sensors with different delay properties are involved.

The random delay in each sensor is modeled by a sequence of Bernoulli random variables, whose parameters represent the delay probabilities. Using an innovation approach, the estimation algorithms are derived without requiring the knowledge of the signal state-space model, but only the covariance functions of the processes involved in the observation equation, as well as the delay probabilities in each sensor. To measure the performance of the estimators, the filtering and smoothing error covariance matrices are also calculated.

The generalization of the current model to a more general case of possibly longer δ sampling delay will require the introduction, for $i = 1, \dots, m$, and $d = 0, \dots, \delta$, of sequences $\{\gamma_k^{(d)i};\ k > d\}$ of independent Bernoulli random variables with $\sum_{d=0}^{\min\{k-1,\delta\}} \gamma_k^{(d)i} = 1$; so, the available measurements to estimate the signal can be described by

$$y_k^i = \sum_{d=0}^{\min(k-1,\delta)} \gamma_k^{(d)i} \widetilde{y}_{k-d}^i, \quad k \geqslant 1,\ i = 1, \dots, m.$$

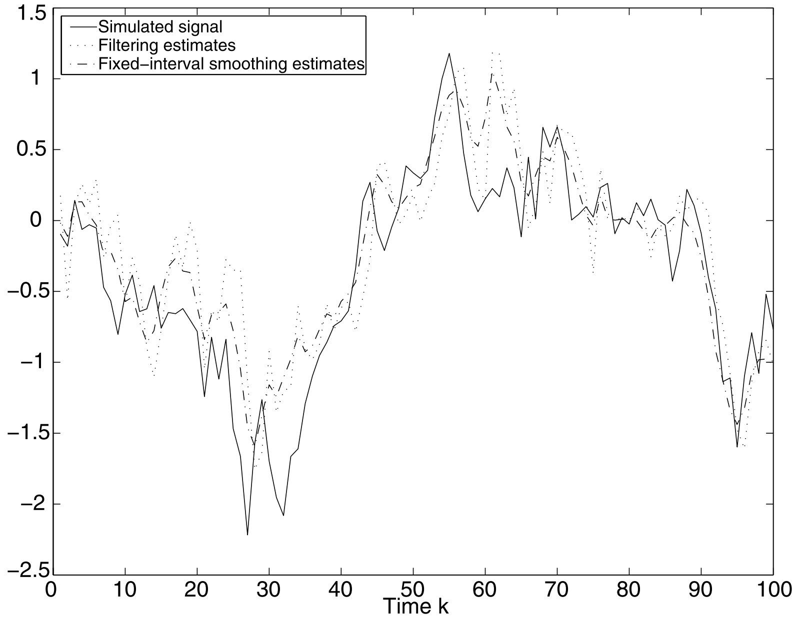


Fig. 5. Simulated signal, z_k, filtering estimates, $\widehat{z}_{k/k}$, and fixed-interval smoothing estimates, $\widehat{z}_{k/100}$, for the values $p^1 = 0.1$ and $p^2 = 0.3$.

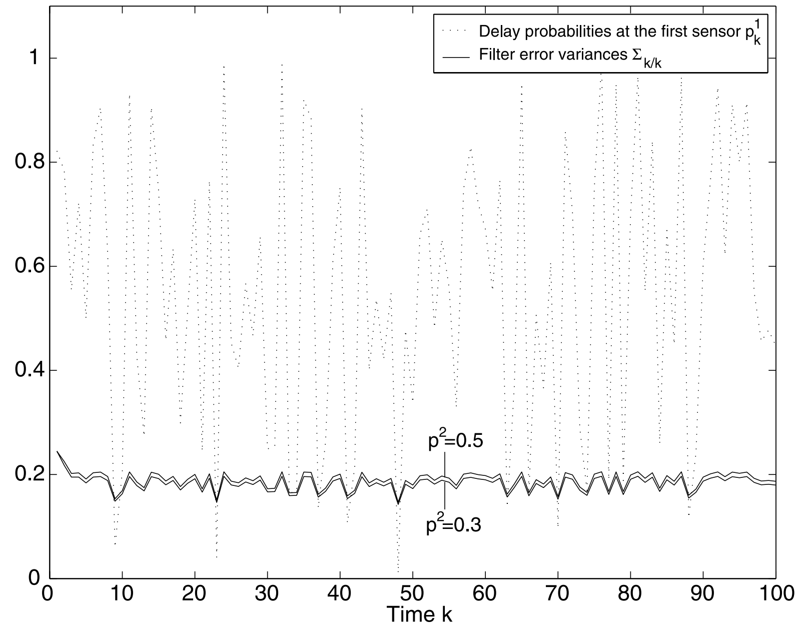


Fig. 6. Filtering error variances for time-variant probabilities p^1_k, when $p^2 = 0.3$ and $p^2 = 0.5$.

The linear least-squares estimators for the case of multiple sampling delays can be derived by following a similar framework to that used in this paper.

To illustrate the theoretical results established in this paper, a simulation example is presented, in which the proposed algorithms are applied to estimate a signal from randomly delayed observations coming from two sensors with different delay characteristics.

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